$\mathcal{L}_1$ Adaptive Control of Event-Triggered Networked Systems

Xiaofeng Wang and Naira Hovakimyan

***Abstract*— This paper studies the implementation of $\mathcal{L}_1$ adaptive controller over real-time networks using event-triggering. Event-triggering schedules the data transmission dependent upon errors exceeding certain threshold. We provide event-triggering schemes to characterize the time instants of data transmissions, while using $\mathcal{L}_1$ adaptive controller in the feedback loop. Lower bounds on transmission periods are provided. We show that with the proposed event-triggering schemes the states and the input in the networked system can be arbitrarily close to those of a stable reference system by increasing the rate of adaptation and the transmission frequency.**

I. Introduction

With the progress in digital technology the feedback loops in a wide range of applications are closed via real-time communication networks for the advantages of lower cost, ease of maintenance and diagnosis, and great flexibility. The introduction of real-time networks raises new challenges regarding the impact that communication has on the system performance. Communication, especially wireless communication, takes place over a digital network, which means that information is transmitted in discrete time rather than continuous-time. Moreover, because all real-time networks have limited bandwidth, the information transmission has to be scheduled in an appropriate manner for a proper operation of the control system. This paper studies the impact of a real-time communication network on the performance of $\mathcal{L}_1$ adaptive controller [1] using an event-triggering technique. Event-triggering has the data transmitted only when "needed". By "needed", it means that some error exceeds certain threshold. Empirical evidence has suggested that event-triggering can largely reduce usage of computational/communication resources. Moreover, it can dynamically adjust the task periods in response to external disturbances [2], [3]. Event-triggering has been studied in [2], [3], [4], [5], [6], [7]. Among these, [2], [3], [4], [5] studied single processor systems and [6], [7], [8] considered distributed implementation. These results are limited to event-triggered feedback in systems without uncertainties.

This paper provides event-triggering schemes to characterize the time instants of data transmissions in uncertain systems with $\mathcal{L}_1$ adaptive controller being in the feedback path. We show that with the proposed event-triggering schemes the states and the input in the networked system can be arbitrarily close to those of a stable reference system by increasing the rate of adaptation and the transmission frequency. We also provide guaranteed lower bounds on transmission periods generated by our scheme and verify those in simulations.

This paper is organized as follows. Section II gives the problem formulation. Section III presents the event-triggering scheme that has guaranteed performance. Simulations results are shown in Section IV. Finally, some discussion is stated in section V.

Both authors are with the Department of Mechanical Science and Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801; e-mail: wangx,nhovakim@illinois.edu. Research is supported by AFOSR under Contract No.FA9550-09-1-0265.

II. Problem Formulation

Notation: We denote by $\mathbb{R}^n$ the n-dimension real vector space and by $\mathbb{R}^+$ the real positive numbers. We also use $\mathbb{R}_0^+ = \mathbb{R}^+ \cup \{0\}$. $\|\cdot\|$ is the Euclidean norm of a vector. $\|\cdot\|_{\mathcal{L}_1}$ and $\|\cdot\|_{\mathcal{L}_\infty}$ are the $\mathcal{L}_1$ and $\mathcal{L}_\infty$ norm of a function, respectively. The truncated $\mathcal{L}_\infty$ norm of a function $x : \mathbb{R}_0^+ \to \mathbb{R}^n$ is defined as $\|x_\tau\|_{\mathcal{L}_\infty} \triangleq \sup_{0\le t\le\tau} \|x(t)\|$. We also use $\vee$ to denote the logical operator OR, where $E_1 \vee E_2$ is true when either E_1 or E_2 is true. $\bar{\cdot}$ denotes the logical operator NOT, where $\overline{E}$ is true when E is false. The Laplace transform of a function $x(t)$ is denoted by $x(s)$. For a function $x : \mathbb{R} \to \mathbb{R}^n$, We denote $x(t^+) = \lim_{\rho\to t^+} x(\rho)$ and $x(t^-) = \lim_{\rho\to t^-} x(\rho)$.

This paper considers an implementation of $\mathcal{L}_1$ adaptive controller in networked systems using event-triggering technique. As shown in Figure 1, in such a system, the plant transmits the state to the controller through a real-time network only when some event occurs. The time instants of transmitting the states can be characterized by a monotonic sequence $\{s_k\}_{k=1}^\infty$, where $s_k \in \mathbb{R}^+$ is the kth time instant of transmitting the state from the plant to the controller. The control input is computed based on the received state using the $\mathcal{L}_1$ adaptive control scheme. The updated control input is transmitted back to the plant only when another event occurs. We use another monotonic sequence $\{r_j\}_{j=1}^\infty$ to characterize the time instants when the input is transmitted back to the plant, where $r_j \in \mathbb{R}^+$ is the jth time instant of transmitting the updated input from the controller to the plant. The control input is held by a zero-order-hold until the next transmission of inputs happens. In Figure 1, solid lines represent continuous signals and dashed lines represent discrete signals. We assume that there is no delay in sampling, data transmission, and computation.

The system under consideration is a single-input-single-output (SISO) system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + b(u(t) - \theta^\top x(t)) \\ y(t) &= c^\top x(t), \quad x(0) = x_0 \end{aligned} \tag{1}$$

where $x : \mathbb{R}_0^+ \to \mathbb{R}^n$ is the state trajectory, $u : \mathbb{R}_0^+ \to \mathbb{R}$ is the control input, $y : \mathbb{R}_0^+ \to \mathbb{R}$ is the system output, $A \in \mathbb{R}^{n\times n}$, $b, c \in \mathbb{R}^n$ are known and $\theta \in \mathbb{R}^n$ is an unknown

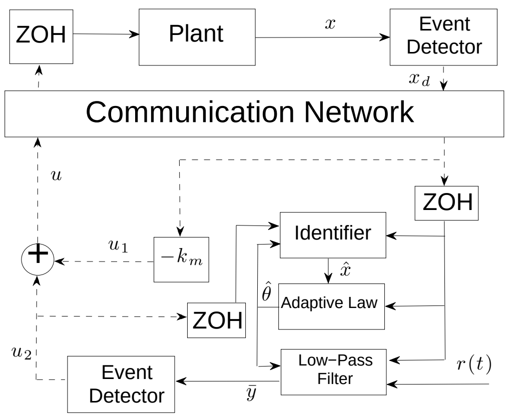

Fig. 1. Networked $\mathcal{L}_1$ adaptive control system

vector that lies in a known compact, convex set $\theta \in \Omega \subseteq \mathbb{R}^n$. We assume that the system is controllable. An $\mathcal{L}_1$ adaptive controller for such a system has the following structure:

$$u(t) = u_1(t) + u_2(t), \tag{2}$$

where $u_1 : \mathbb{R}_0^+ \to \mathbb{R}$ is the nominal feedback signal and $u_2 : \mathbb{R}_0^+ \to \mathbb{R}$ is the adaptive signal. The nominal feedback signal is given by

$$u_1(t) = -k_m^\top x_d(t), \tag{3}$$

where $k_m \in \mathbb{R}^n$ is the feedback gain that renders $A_m \triangleq A - bk_m^\top$ Hurwitz, $x_d : \mathbb{R}_0^+ \to \mathbb{R}^n$ is the latest transmitted state at time t and is piecewise constant. Notice that $x_d(t) = x(s_k)$ for all $t \in [s_k, s_{k+1})$. Therefore, $u_1(t)$ is also piecewise constant. Since A_m is Hurwitz, then given arbitrary $Q > 0$, there exists $P = P^\top > 0$ such that $PA_m + A_m^\top P = -Q$.

To define the adaptive signal, we first need to introduce a passive identifier:

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + b(-k_m^\top \hat{x}(t) + u_2(t) - \hat{\theta}^\top(t) x_d(t)) \\ \hat{y}(t) &= c^\top \hat{x}(t), \quad \hat{x}(0) = x_0 \end{aligned} \tag{4}$$

where $\hat{x} : \mathbb{R}_0^+ \to \mathbb{R}^n, \hat{y} : \mathbb{R}_0^+ \to \mathbb{R}$ are the state and outputs of the identifier, respectively. In equation (4), $\hat{\theta} : \mathbb{R}_0^+ \to \mathbb{R}^n$ is updated according to the adaptive law

$$\dot{\hat{\theta}}(t) = \Gamma \mathbf{Proj}(\hat{\theta}(t), x_d(t)(\hat{x}(t) - x_d(t))^\top Pb), \tag{5}$$

where $\Gamma \in \mathbb{R}^+$ is the adaptation gain and the operator $\mathbf{Proj} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is given by

$$\mathbf{Proj}(\hat{\theta}, x) = \begin{cases} x & \text{if } h(\hat{\theta}) < 0 \\ x & \text{if } h(\hat{\theta}) > 0, \nabla h^\top x \le 0 \\ x - \frac{\nabla h \nabla h^\top x h(\hat{\theta})}{\|\nabla h\|^2} & \text{if } h(\hat{\theta}) > 0, \nabla h^\top x > 0 \end{cases} \tag{6}$$

where $h(\hat{\theta}) = \frac{\hat{\theta}^\top \hat{\theta} - \theta_{\max}^2}{\epsilon_\theta \theta_{\max}^2}$, $\theta_{\max} \in \mathbb{R}^+$ is the norm bound imposed on $\hat{\theta}$, and $\epsilon_\theta \in \mathbb{R}^+$ is the convergence tolerance of the bound. By appropriately choosing $\theta_{\max}$ and ϵ_θ, the operator $\mathbf{Proj}$ ensures $\hat{\theta}(t) \in \Omega$ for all $t \ge 0$. More details on $\mathbf{Proj}$ operator can be found in [9].

Lemma 2.1 ([9]): Given $x, \hat{\theta} \in \mathbb{R}^n$, we have $(\hat{\theta} - \theta)^\top (\mathbf{Proj}(\hat{\theta}, x) - x) \le 0$, where θ is the true value of $\hat{\theta}$.

The adaptive signal u_2 is defined as follows

$$u_2(t) = \bar{y}(r_j) \tag{7}$$

for all $t \in [r_j, r_{j+1})$, where $\bar{y} : \mathbb{R}_0^+ \to \mathbb{R}$ is the output of the low-pass filter

$$\begin{aligned} \dot{\bar{x}}(t) &= \bar{A}\bar{x}(t) + \bar{b}(\hat{\theta}^\top(t) x_d(t) + k_g r(t)) \\ \bar{y}(t) &= \bar{c}^\top \bar{x}(t), \quad \bar{x}(0) = 0 \end{aligned} \tag{8}$$

In equation (8), $\bar{x} : \mathbb{R}_0^+ \to \mathbb{R}^m$ is the state trajectory of this system and $k_g = \frac{1}{c^\top H(0)}$. Let $C(s) = \bar{c}(s\mathbb{I} - \bar{A})^{-1}\bar{b}$. In $\mathcal{L}_1$ adaptive control design, $C(s)$ is chosen as a low-pass filter to filter high frequency signals in the input u. We enforce $C(0) = 1$. Let $H(s) \triangleq (s\mathbb{I} - A_m)^{-1} b$, $G(s) \triangleq H(s)(C(s) - 1)$, $L \triangleq \max_{\theta \in \Omega} \|\theta\|$. The choice of $C(s)$ needs to verify the following upper bound

$$\sigma \triangleq \|G(s)\|_{\mathcal{L}_1} L < 1. \tag{9}$$

Lemma 2.2 ([1]): If (A_m, b) is controllable and $H(s)$ is asymptotically stable, then there exists $c_o \in \mathbb{R}^n$ such that $c_o^\top H(s)$ is minimum phase with relatively degree 1.

In the following discussion, we design event-triggering schemes to characterize the sequences $\{s_k\}_{k=1}^\infty$ and $\{r_j\}_{j=1}^\infty$ such that the the system output $y(t)$ follows the reference input $r(t)$ with quantifiable performance bounds.

III. Event-Triggered Data Transmission

Let $\tilde{x}(t) \triangleq \hat{x}(t) - x(t)$, $\tilde{\theta}(t) \triangleq \hat{\theta}(t) - \theta$, $e(t) \triangleq x_d(t) - x(t)$, and $\bar{e}(t) \triangleq \bar{x}(r_j) - \bar{x}(t), \quad \forall t \in [r_j, r_{j+1})$. Given $\xi \in \mathbb{R}^+$, we define four logic rules:

$$E_1 : \|e(t)\| \le \xi \tag{10}$$

$$E_2 : \|x_d(t) e^\top(t) Pb\| \le \xi^2 \tag{11}$$

$$E_3 : \|\bar{e}(t)\| \le \xi \tag{12}$$

$$E_4 : \text{The controller receives } x_d. \tag{13}$$

Remark 3.1: The parameter ξ in E_1, E_2 and E_3 does not have to be the same. We can use ξ_1, ξ_2, ξ_3 instead. For notational convenience, we use one parameter ξ in this paper.

The plant uses the violation of inequality (10) or (11) to trigger the transmission of the state information, x_d. In other words, when $\overline{E}_1 \vee \overline{E}_2$ is true, the plant transmits the state to the controller. s_k is the moment when $\overline{E}_1 \vee \overline{E}_2$ is true for the kth time. Notice that right after the plant transmits the state, $e(t)$ becomes zero. Then (10) and (11) are trivially satisfied and $\overline{E}_1 \vee \overline{E}_2$ becomes false. When (12) is violated or the controller receives a transmitted state from the plant, the controller transmits the control input u back to the plant. In other words, the input is transmitted back to the plant, when $\overline{E}_3 \vee E_4$ is true. r_j is the moment when $\overline{E}_3 \vee E_4$ is true for the jth time. Notice that $\{s_k\}_{k=1}^\infty$ is a subsequence of $\{r_j\}_{j=1}^\infty$. Right after the controller sends $u(t)$ to the plant, $\bar{e}(t) = 0$ holds and (12) is trivially satisfied.

Similar to [1], we consider the following closed-loop reference system with its controller

$$\begin{aligned} \dot{x}_{\text{ref}}(t) &= Ax_{\text{ref}}(t) + b(u_{\text{ref}}(t) - \theta^\top x_{\text{ref}}(t)) \\ y_{\text{ref}}(t) &= c^\top x_{\text{ref}}(t), \quad x_{\text{ref}}(0) = x_0\,, \end{aligned} \tag{14}$$

$$\begin{aligned}\dot{\bar{x}}_{\rm ref}(t) &= \bar{A}\bar{x}_{\rm ref}(t) + \bar{b}(\theta^\top x_{\rm ref}(t) + k_g r(t)) \\ u_{\rm ref}(t) &= \bar{c}^\top \bar{x}_{\rm ref}(t) - k_m^\top x_{\rm ref}(t), \quad \bar{x}(0) = 0 \end{aligned} \quad (15)$$

In frequency domain it can be equivalently rewritten:

$$\begin{aligned} x_{\rm ref}(s) &= H(s)C(s)k_g r(s) + (s\mathbb{I} - A_m)^{-1}x_0 \\ &\quad + H(s)(C(s)-1)\theta^\top x_{\rm ref}(s) \\ y_{\rm ref}(s) &= c^\top x_{\rm ref}(s) \quad (16) \\ u_{\rm ref}(s) &= C(s)(\theta^\top x_{\rm ref}(s) + k_g r(s) - k_m^\top x_{\rm ref}(s) \end{aligned}$$

Lemma 3.1 ([1]): Consider the reference system in (17). If $\sigma < 1$, where σ is defined in (9), then this reference system is BIBO stable with respect to $r(t)$, x_0. Moreover, $\|x_{\rm ref}\|_{\mathcal{L}_\infty} \leq \frac{\|H(s)k_g C(s)\|_{\mathcal{L}_1}}{1-\|G(s)\theta^\top\|_{\mathcal{L}_1}}\|r\|_{\mathcal{L}_\infty} + \frac{\|(s\mathbb{I}-A_m)^{-1}\|_{\mathcal{L}_1}}{1-\|G(s)\theta^\top\|_{\mathcal{L}_1}}\|x_0\|$.

To establish the closeness between the networked system and the reference system, we first need the following lemma to show the connection between the networked system and the identifier.

Lemma 3.2: Consider the system (1) – (8). Given a positive constant $\xi \in \mathbb{R}^+$, if (10), (11), and (12) hold, then

$$\|\tilde{x}\|_{\mathcal{L}_\infty} \leq \max\left\{\sqrt{\frac{\bar{L}}{\lambda_{\min}(P)\Gamma}},\ \xi\chi\right\}, \quad (17)$$

where

$$\bar{L} \triangleq \max_{\theta\in\Omega} 4\theta^\top\theta \quad (18)$$

$$\chi \triangleq \frac{\|Pbk_m^\top\|+L\|Pb\|+\sqrt{(\|Pbk_m^\top\|+L\|Pb\|)^2+4L\lambda_{\min}(Q)}}{\lambda_{\min}(Q)\lambda_{\min}(P)/\lambda_{\max}(P)},$$

and

$$\|\hat{x}\|_{\mathcal{L}_\infty} \leq \phi_1, \quad \|x\|_{\mathcal{L}_\infty} \leq \phi_2 \quad (19)$$

with

$$\begin{aligned}\phi_1 &\triangleq \frac{\sigma\max\left\{\xi\chi,\sqrt{\frac{\bar{L}}{\lambda_{\min}(P)\Gamma}}\right\}+\|(s\mathbb{I}-A_m)^{-1}\|_{\mathcal{L}_1}\|x_0\|}{1-\sigma} \\ &\quad + \frac{\|H(s)C(s)k_g\|_{\mathcal{L}_1}\|r\|_{\mathcal{L}_\infty}+\|H(s)\bar{c}^\top\|_{\mathcal{L}_1}\xi}{1-\sigma} \quad (20) \\ \phi_2 &\triangleq \phi_1 + \max\left\{\sqrt{\frac{\bar{L}}{\lambda_{\min}(P)\Gamma}}, \xi\chi\right\}, \quad (21)\end{aligned}$$

respectively.

With Lemma 3.2, we are able to provide uniform bounds on the difference of the states and the inputs between the networked system and the reference system.

Theorem 3.3: Consider the networked system defined by (1) – (8). Given a positive constant ξ, if (10), (11), and (12) hold, then we have

$$\|x_{\rm ref} - x\|_{\mathcal{L}_\infty} \leq \max\{\xi\gamma_1 + \tfrac{\gamma_2}{\sqrt{\Gamma}}, \gamma_3\xi\} \quad (22)$$

$$\|u_{\rm ref} - u\|_{\mathcal{L}_\infty} \leq \max\{\xi\gamma_4 + \tfrac{\gamma_5}{\sqrt{\Gamma}}, \gamma_6\xi\} \quad (23)$$

where

$$\begin{aligned}\gamma_1 &\triangleq \frac{\|H(s)\bar{c}^\top\|_{\mathcal{L}_1}+\|G(s)k_m^\top\|_{\mathcal{L}_1}}{1-\sigma} \quad (24) \\ \gamma_2 &\triangleq \frac{\|C(s)\|_{\mathcal{L}_1}}{1-\sigma}\sqrt{\frac{\bar{L}}{\lambda_{\min}(P)}} \\ \gamma_3 &\triangleq \gamma_1 + \frac{\chi\|C(s)\|_{\mathcal{L}_1}}{1-\sigma} \\ \gamma_4 &\triangleq \|(1-C(s))k_m^\top\|_{\mathcal{L}_1} + \|\bar{c}\| + \|C(s)\theta^\top - k_m^\top\|_{\mathcal{L}_1}\gamma_1 \\ \gamma_5 &\triangleq \left\|\frac{C(s)c_o^\top}{c_o^\top H(s)}\right\|_{\mathcal{L}_1}\sqrt{\frac{\bar{L}}{\lambda_{\min}(P)}} + \|C(s)\theta^\top - k_m^\top\|_{\mathcal{L}_1}\gamma_2 \\ \gamma_6 &\triangleq \gamma_4 + \chi\left(\left\|\frac{C(s)c_o^\top}{c_o^\top H(s)}\right\|_{\mathcal{L}_1} + \frac{\|C(s)\theta^\top - k_m^\top\|_{\mathcal{L}_1}\|C(s)\|_{\mathcal{L}_1}}{1-\sigma}\right).\end{aligned}$$

Remark 3.2: According to equations (22) and (23), we can increase Γ and decrease ξ to diminish the difference between the reference model and the networked system. Note that if ξ is zero, we can recover the uniform bounds in [1].

Remark 3.3: Notice that if $C(s) = 1$, then the closed-loop system is equivalent to MRAC. In this case, γ_5 will be unbounded because the term $\frac{C(s)c_o^\top}{c_o^\top H(s)}$ in γ_5 is improper. Therefore, for the control signal in MRAC, a uniform performance bound cannot be obtained with the above described event-triggering scheme.

Theorem 3.3 provides unform performance bounds for the signals in the closed-loop system using the proposed event-triggering scheme. Next we derive the bounds on the transmission periods generated by this scheme. Recall that $s_k \in \mathbb{R}^+$ is the kth transmission time instant from the plant to the controller, $r_j \in \mathbb{R}^+$ is the jth transmission time instant from the controller to the plant, and $\{s_k\}_{k=1}^\infty$ is subsequence of $\{r_j\}_{j=1}^\infty$. Let $r_i^k \in (s_k, s_{k+1})$ be the ith time instant when E_3 is violated over (s_k, s_{k+1}). Therefore, $\{r_j\}_{j=1}^\infty = \{s_k\}_{k=1}^\infty \cup \{r_i^k\}_{k=1,i=1}^{\infty,m_k}$, where $m_k \in \mathbb{N}$ is the number of times E_3 is violated over (s_k, s_{k+1}). Let $r_0^k \triangleq s_k$.

Corollary 3.4: Consider the networked system defined by (1) – (8). Given $\xi \in \mathbb{R}^+$, if the transmission of states from the plant to the controller is triggered when $\overline{E}_1 \vee \overline{E}_2$ is true, and the transmission of inputs from the controller to the plant is triggered when $\overline{E}_3 \vee E_4$ is true, then

$$\forall k \in \mathbb{N}, \quad s_{k+1} - s_k \geq \alpha_1 > 0, \quad (25)$$

$$\forall, i \in \{0, \cdots, m_k\}, \forall k \in \mathbb{N}, \quad r_{i+1}^k - r_i^k \geq \alpha_2 > 0, \quad (26)$$

where $\alpha_1, \alpha_2 \in \mathbb{R}^+$ are defined by

$$\begin{aligned}\alpha_1 &\triangleq \frac{\xi}{\psi_1}\min\{1, \frac{\xi}{\phi_2\|Pb\|}\} \quad (27) \\ \alpha_2 &\triangleq \frac{\xi}{\psi_2} \quad (28) \\ \psi_1 &\triangleq \|b\|\|C(s)k_g\|_{\mathcal{L}_1}\|r\|_{\mathcal{L}_\infty} + \|b\|\|\bar{c}\|\xi + \quad (29) \\ &\quad (\|A - b\theta^\top\| + \|b\|(L\|C(s)\|_{\mathcal{L}_1} + \|k_m\|))\phi_2 \\ \psi_2 &\triangleq (L\|\bar{A}\|\|(s\mathbb{I}-\bar{A})^{-1}\bar{b}\|_{\mathcal{L}_1} + L\|\bar{b}\|)\phi_2 \\ &\quad + (\|\bar{A}\|\|(s\mathbb{I}-\bar{A})^{-1}\bar{b}\|_{\mathcal{L}_1}\|k_g\| + \|\bar{b}k_g\|)\|r\|_{\mathcal{L}_\infty} \quad (30)\end{aligned}$$

and ϕ_2 is given in (21).

Remark 3.4: Corollary 3.4 shows that the transmission periods from the plant to the controller are always greater than a positive constant α_1. We also show that the periods of the violation of E_3 are greater than α_2. However, we cannot claim that the transmission periods from the controller to the plant are greater than a positive constant. This is because $r_{m_k}^k$, which is the last time instant when E_3 is violated during (s_k, s_{k+1}), may be arbitrarily close to s_{k+1}.

Remark 3.5: The bounds in (27) and (28) can be used for a periodic implementation of networked $\mathcal{L}_1$ adaptive control. Let $l = \left\lfloor \frac{\alpha_1}{\alpha_2} \right\rfloor$. Assume that the transmission period from the plant to the controller is α_1 and the period from the controller to the plant is $\frac{\alpha_1}{l+1}$. It is easy to see that $E_1 - E_4$ are satisfied using this periodic model, which means the uniform bounds in equations (22) and (23) are still valid.

Remark 3.6: The lower bound on the transmission periods are determined by three parameters, Γ, ξ, and $\|r\|_{\mathcal{L}_\infty}$. From

(20) and (21), we know ϕ_2 increases as ξ and $\|r\|_{\mathcal{L}_\infty}$ increase and Γ decreases. Therefore, according to (29) and (30), ψ_1 and ψ_2 increase as ξ and $\|r\|_{\mathcal{L}_\infty}$ increase and Γ decreases. Then based on (27) and (28), α_1 and α_2 increase, when Γ and ξ increase and $\|r\|_{\mathcal{L}_\infty}$ decreases. One thing worth mentioning is that for fixed ξ, Γ will not affect the value of α_1 and α_2 if it is large enough, because in that case the term $\max\left\{\sqrt{\frac{\bar{L}}{\lambda_{\min}(P)\Gamma}},\ \xi\chi\right\}$ in ϕ_2 is dominated by $\xi\chi$.

IV. An Illustrative Example

This section illustrates how the event-triggering scheme works with $\mathcal{L}_1$ adaptive controller. We use the example from [1]. Let

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -1.4 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad c = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$k_m = \begin{bmatrix} 1 \\ 1.6 \end{bmatrix}, \quad \theta = \begin{bmatrix} 4 \\ -4.5 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Further, $\Omega = [-10, 10] \times [-10, 10]$, resulting in $L = 20$. The adaptation gain is $\Gamma = 10000$, and the low-pass filter $C(s)$ is $C(s) = \frac{160}{s+160}$, which ensures $\sigma = \|G(s)\|_{\mathcal{L}_1} L < 1$. We set $\bar{A} = -160$, $\bar{b} = 10$, and $\bar{c} = 16$. Also we set $\xi = 0.5$.

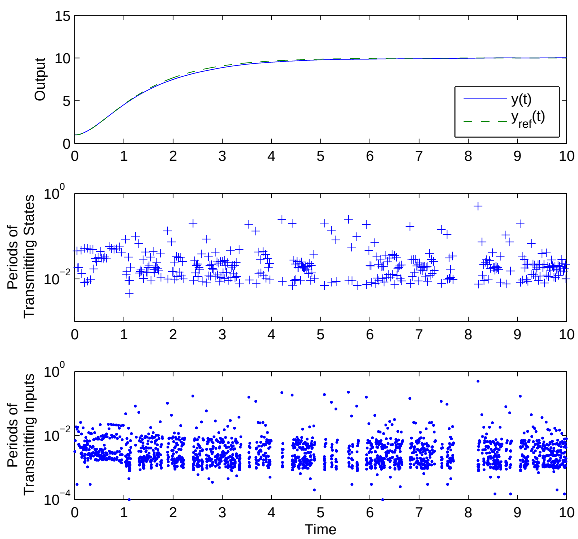

Fig. 2. An event-triggered networked system with $r(t) \equiv 10$

In this simulation, the data transmission from the plant to the controller is triggered when $\overline{E}_1 \vee \overline{E}_2$ is true. The transmission of the control input is triggered when $\overline{E}_3 \vee E_4$ is true. We first set $r(t) \equiv 10$. The simulation results are shown in Figure 2. The top plot in Figure 2 shows the history of the outputs of the networked system and the reference system. From the plot, we can see that these two curves are almost identical. It implies that the signals in the networked system can be very close to those in the reference model. The middle plot in Figure 2 shows the periods of transmitting states from the plant to the controller generated by the event-triggering scheme. Notice that the periods vary a lot. It demonstrates the ability of event-triggering in adjusting the periods in response to the variations in the states. Also note that those periods are always bounded away from zero. The minimum/average period is 0.0046/0.0306 seconds, respectively. The bottom plot shows the periods of transmitting inputs versus time. These periods also vary in a wide range with the minimum and average periods of 10^{-4} and 0.0057 seconds, respectively. It is because the periods of transmitting inputs are always less than that of transmitting states, since $\{s_k\}_{k=1}^{\infty}$ is a subsequence of $\{r_j\}_{j=1}^{\infty}$.

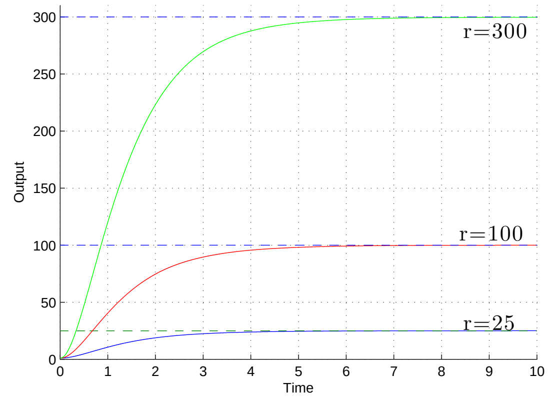

Fig. 3. The outputs of networked systems with $r(t) = 25, 100, 300$

Next we verify whether the event-triggered $\mathcal{L}_1$ adaptive controller retains its scaling properties from [1]. In this simulation, $r(t)$ is set to be 25, 100, and 300. Figures 3 and 4 plot the histories of outputs and inputs of the systems associated with different tracking signals. From these plots, we can see that the networked system has similar scaling properties with respect to the reference signals. Another fact worth mentioning is that as $r(t)$ is changing from 25 to 300, the average period of transmitting states (inputs) was reduced from 0.0199 to 4×10^{-4} (from 0.0049 to 3×10^{-4}), which is consistent with Remark 3.6.

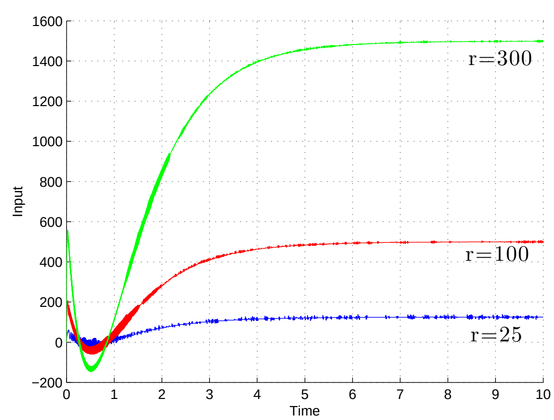

Fig. 4. The inputs of networked systems with $r(t) = 25, 100, 300$

V. Discussion

This paper presents the implementation of $\mathcal{L}_1$ adaptive controller over networks. Event-triggering schemes are proposed to ensure system performance. Lower bounds on the transmission periods are also provided. Although this paper focuses on the real-time issue in networked control, our framework and results can also be used to quantify the performance in case of transmission delays and quantization effects. For example, when quantization is considered, $x_d(t) = q(x(s_k))$ for all $t \in [s_k, s_{k+1})$, where $q : \mathbb{R}^n \to \mathbb{R}^n$ is the quantizer. We can use the real-time constraints in E_1 to E_4 as the guidance to design a quantizer and/or derive bounds on delays. The lower bound on the periods derived in this paper can be used as a reference parameter when designing the network. It can also be used to discretize the controller. This can be very helpful, especially when the computation ability of the controller is limited.

APPENDIX

Proof: [Proof of Lemma 3.2] We first derive the upper bound on $\|\tilde{x}\|_{\mathcal{L}_\infty}$. For notational convenience, we drop the argument t, if it is clear from the context. By (1), (4),

$$\dot{\tilde{x}} = \dot{\hat{x}} - \dot{x} = A_m\tilde{x} + bk_m^\top e - b\theta^\top e - b\tilde{\theta}^\top x_d.$$

Let $V(\tilde{x}, \tilde{\theta}) = \tilde{x}^\top P\tilde{x} + \tilde{\theta}^\top\Gamma^{-1}\tilde{\theta}$. Compute $\dot{V}(t)$:

$$\begin{aligned}\dot{V} &= 2\tilde{x}^\top P\dot{\tilde{x}} + 2\tilde{\theta}^\top\Gamma^{-1}\dot{\tilde{\theta}} \\ &= -\tilde{x}^\top Q\tilde{x} + 2\tilde{x}^\top Pbk_m^\top e - 2\tilde{x}^\top Pb\theta^\top e \\ &\quad -2e^\top Pb\tilde{\theta}^\top x_d - 2(\hat{x} - x_d)^\top Pb\tilde{\theta}^\top x_d + 2\tilde{\theta}^\top\Gamma^{-1}\dot{\tilde{\theta}}.\end{aligned}$$

The adaptive law from (5) along with the property stated in Lemma 2.1 leads to

$$\dot{V} \le -\tilde{x}^\top Q\tilde{x} + 2\tilde{x}^\top Pbk_m^\top e - 2\tilde{x}^\top Pb\theta^\top e - 2e^\top Pb\tilde{\theta}^\top x_d.$$

Using the upper bounds from (10) and (11) along with the fact $\|\tilde{\theta}\| \le \|\hat{\theta}\| + \|\theta\| \le 2L$ in this inequality implies

$$\begin{aligned}&\dot{V} \le -\lambda_{\min}(Q)\|\tilde{x}\|^2 + 2\tilde{x}^\top Pb(k_m^\top e - \theta^\top e) - 2e^\top Pb\tilde{\theta}^\top x_d \\ &\le -\lambda_{\min}(Q)\|\tilde{x}\|^2 + 2\|\tilde{x}\|\|Pbk_m^\top\|\xi + 2L\|\tilde{x}\|\|Pb\|\xi + 4L\xi^2.\end{aligned}$$

This inequality implies that for any $t \ge 0$, either $\dot{V} \le 0$ or $\|\tilde{x}(t)\| \le \xi\chi$ holds, where χ is defined in (18). We therefore have eithor $\tilde{x}^\top(t)\tilde{x}(t) \le \frac{V(0)}{\lambda_{\min}(P)} \le \frac{\bar{L}}{\lambda_{\min}(P)\Gamma}$ or $\|\tilde{x}(t)\| \le \xi\chi$ for all $t \ge 0$, which implies (17).

We now show the upper bound on $\|\hat{x}\|_{\mathcal{L}_\infty}$. By (17),

$$\|x_\tau\|_{\mathcal{L}_\infty} \le \|\hat{x}_\tau\|_{\mathcal{L}_\infty} + \max\left\{\sqrt{\tfrac{\bar{L}}{\lambda_{\min}(P)\Gamma}}, \xi\chi\right\}. \qquad (31)$$

Projection based adaptation in (5) ensures that $\hat{\theta}(t) \in \Omega$ for all $t \ge 0$, and therefore $\|\hat{\theta}(t)\| \le L$ for all $t \ge 0$. Also notice that $\|c^\top x_{d\tau}\|_{\mathcal{L}_\infty} \le \|c^\top x_\tau\|_{\mathcal{L}_\infty}$ holds since $x_d(t)$ is the sampled signal of $x(t)$. Combining this with the upper bound in (31) yields

$$\begin{aligned}\|\hat{r}_\tau\|_{\mathcal{L}_\infty} &\le L\|x_{d\tau}\|_{\mathcal{L}_\infty} \le L\|x_\tau\|_{\mathcal{L}_\infty} \qquad (32)\\ &\le L\left(\|\hat{x}_\tau\|_{\mathcal{L}_\infty} + \max\left\{\sqrt{\tfrac{\bar{L}}{\lambda_{\min}(P)\Gamma}}, \xi\chi\right\}\right),\end{aligned}$$

where $\hat{r} : \mathbb{R}_0^+ \to \mathbb{R}$ is defined by $\hat{r}(t) \triangleq \hat{\theta}^\top(t)x_d(t)$. Taking the Laplace transform of (4) and (8) yields

$$\hat{x}(s) = H(s)u_2(s) - H(s)\hat{r}(s) + (s\mathbb{I} - A_m)^{-1}x_0 \qquad (33)$$

$$u_2(s) = C(s)(\hat{r}(s) + k_g r(s)) + \bar{c}^\top\bar{e}(s) \qquad (34)$$

Therefore

$$\begin{aligned}\hat{x}(s) &= H(s)(C(s) - 1)\hat{r}(s) + H(s)C(s)k_g r(s) \\ &\quad +H(s)\bar{c}^\top\bar{e}(s) + (s\mathbb{I} - A_m)^{-1}x_0 \qquad (35)\end{aligned}$$

holds, which implies

$$\begin{aligned}\|\hat{x}_\tau\|_{\mathcal{L}_\infty} &\le \|G(s)\|_{\mathcal{L}_1}\|\hat{r}_\tau\|_{\mathcal{L}_\infty} + \|H(s)C(s)k_g\|_{\mathcal{L}_1}\|r_\tau\|_{\mathcal{L}_\infty} \\ &\quad +\|H(s)\bar{c}^\top\|_{\mathcal{L}_1}\|\bar{e}_\tau\|_{\mathcal{L}_\infty} + \|(s\mathbb{I} - A_m)^{-1}\|_{\mathcal{L}_1}\|x_0\|.\end{aligned}$$

Using the upper bound from (32) in this inequality implies

$$\begin{aligned}\|\hat{x}_\tau\|_{\mathcal{L}_\infty} &\le \sigma\|\hat{x}_\tau\|_{\mathcal{L}_\infty} + \sigma\max\left\{\sqrt{\tfrac{\bar{L}}{\lambda_{\min}(P)\Gamma}}, \xi\chi\right\} \\ &\quad +\|H(s)\bar{c}^\top\|_{\mathcal{L}_1}\|\bar{e}_\tau\|_{\mathcal{L}_\infty} + \|(s\mathbb{I} - A_m)^{-1}\|_{\mathcal{L}_1}\|x_0\| \\ &\quad +\|H(s)C(s)k_g\|_{\mathcal{L}_1}\|r_\tau\|_{\mathcal{L}_\infty},\end{aligned}$$

where σ is defined by equation (9). Since $\sigma < 1$, we know, from the equation above, that $\|\hat{x}_\tau\|_{\mathcal{L}_\infty} \le \phi_1$ holds. From (12) we have $\|\bar{e}(t)\| \le \xi$ for all $t \ge 0$. Therefore, $\|\bar{e}_\tau\|_{\mathcal{L}_\infty} \le \xi$. With $r \in \mathcal{L}_\infty$, we know the bound on $\|\hat{x}_\tau\|_{\mathcal{L}_\infty}$ is bounded and uniform, i.e. independent of τ. So we have $\hat{x} \in \mathcal{L}_\infty$. Therefore, $x \in \mathcal{L}_\infty$ also holds by (31). ■

Proof: [Proof of Theorem 3.3] Let $\tilde{x}_{\text{ref}}(t) = x_{\text{ref}}(t) - x(t)$, $\tilde{u}_{\text{ref}}(t) = u_{\text{ref}}(t) - u(t)$, $z(t) = \bar{x}_{\text{ref}}(t) - \bar{x}(t)$. From (1) and (14), we have

$$\begin{aligned}\dot{\tilde{x}}_{\text{ref}}(t) &= A\tilde{x}_{\text{ref}}(t) + b(\tilde{u}_{\text{ref}}(t) - \theta^\top\tilde{x}_{\text{ref}}(t)) \\ \tilde{x}_{\text{ref}}(0) &= 0. \qquad (36)\end{aligned}$$

From (2), (3), (8), and (15), one can obtain

$$\dot{z}(t) = \bar{A}z(t) + \bar{b}(\theta^\top x_{\text{ref}}(t) - \hat{\theta}^\top(t)x_d(t)) \qquad (37)$$

$$\tilde{u}_{\text{ref}}(t) = \bar{c}^\top(z(t) - \bar{e}(t)) - k_m^\top(\tilde{x}_{\text{ref}}(t) - e(t)) \qquad (38)$$

Substituting (38) into (36) yields

$$\begin{aligned}\dot{\tilde{x}}_{\text{ref}}(t) &= A_m\tilde{x}_{\text{ref}}(t) + \qquad (39)\\ &\quad b(\bar{c}^\top z(t) - \bar{c}^\top\bar{e}(t) + k_m^\top e(t) - \theta^\top\tilde{x}_{\text{ref}}(t)).\end{aligned}$$

Taking the Laplace transform of (37) and (39), we obtain

$$z(s) = (s\mathbb{I} - \bar{A})^{-1}\bar{b}(\theta^\top x_{\text{ref}}(s) - \hat{r}(s))$$

$$\tilde{x}_{\text{ref}}(s) = H(s)(\bar{c}^\top z(s) - \bar{c}^\top\bar{e}(s) + k_m^\top e(s) - \theta^\top\tilde{x}_{\text{ref}}(s))$$

where $\hat{r}(s)$ is the Laplace transform of $\hat{r}(t) = \hat{\theta}^\top(t)x_d(t)$.

The equations above imply that

$$\begin{aligned}\tilde{x}_{\text{ref}}(s) &= H(s)(C(s) - 1)\theta^\top\tilde{x}_{\text{ref}}(s) \\ &\quad +H(s)C(s)(\theta^\top x(s) - \hat{r}(s))) \\ &\quad +H(s)(k_m^\top e(s) - \bar{c}^\top\bar{e}(s)). \qquad (40)\end{aligned}$$

From (1) and (4) we have

$$\dot{\tilde{x}}(t) = A_m\tilde{x}(t) + bk_m^\top e(t) + b(\theta^\top x(t) - \hat{r}(t)), \quad \tilde{x}(0) = 0$$

Taking Laplace transform of the above equation yields

$$\tilde{x}(s) = H(s)(k_m^\top e(s) + \theta^\top x(s) - \hat{r}(s)), \qquad (41)$$

and therefore $H(s)(\theta^\top x(s) - \hat{r}(s)) = \tilde{x}(s) - H(s)k_m^\top e(s)$ holds. Substituting this equation into (40) implies

$$\begin{aligned}\tilde{x}_{\text{ref}}(s) &= H(s)(C(s) - 1)\theta^\top\tilde{x}_{\text{ref}}(s) \\ &\quad +C(s)(\tilde{x}(s) - H(s)k_m^\top e(s)) \\ &\quad +H(s)(k_m^\top e(s) - \bar{c}^\top\bar{e}(s)), \qquad (42)\end{aligned}$$

which means that for any $\tau \geq 0$

$$\|\tilde{x}_{\mathrm{ref}_\tau}\|_{\mathcal{L}_\infty} \leq \|G(s)\theta^\top\|_{\mathcal{L}_1}\|\tilde{x}_{\mathrm{ref}_\tau}\|_{\mathcal{L}_\infty} + \|H(s)\bar{c}^\top\|_{\mathcal{L}_1}\|\bar{e}_\tau\|_{\mathcal{L}_\infty} + \|C(s)\|_{\mathcal{L}_1}\|\tilde{x}_\tau\|_{\mathcal{L}_\infty} + \|(C(s)-1)H(s)k_m^\top\|_{\mathcal{L}_1}\|e_\tau\|_{\mathcal{L}_\infty}$$

holds and therefore, with equation (9)

$$\|\tilde{x}_{\mathrm{ref}_\tau}\|_{\mathcal{L}_\infty} \leq \frac{\|H(s)\bar{c}^\top\|_{\mathcal{L}_1}\|\bar{e}_\tau\|_{\mathcal{L}_\infty} + \|C(s)\|_{\mathcal{L}_1}\|\tilde{x}_\tau\|_{\mathcal{L}_\infty}}{1-\sigma} + \frac{\|G(s)k_m^\top\|_{\mathcal{L}_1}\|e_\tau\|_{\mathcal{L}_\infty}}{1-\sigma}. \quad (43)$$

From (10) and (12), we have

$$\|e(t)\| \leq \xi, \quad \|\bar{e}(t)\| \leq \xi \quad (44)$$

for all $t \geq 0$. From Lemma 3.2 we know that (17) holds. Substituting (17) and (44) into (43) yields $\|\tilde{x}_{\mathrm{ref}_\tau}\|_{\mathcal{L}_\infty} \leq \max\left\{\xi\gamma_1 + \frac{\gamma_2}{\sqrt{\Gamma}}, \gamma_3\xi\right\}$. Since this upper bound is independent of τ, the bound in (22) holds.

Consider $\tilde{u}_{\mathrm{ref}}(t)$ now. From (2), (3), (7), and (8), we have

$$u(s) = C(s)(\hat{r}(s) + k_g r(s)) + \bar{c}^\top\bar{e}(s) - k_m^\top(x(s) + e(s)).$$

Combining this equation and (17) implies

$$\tilde{u}_{\mathrm{ref}}(s) = C(s)(\theta^\top x(s) - \hat{r}(s)) - k_m^\top\tilde{x}_{\mathrm{ref}}(s) - \bar{c}^\top\bar{e}(s) + k_m^\top e(s) + C(s)\theta^\top\tilde{x}_{\mathrm{ref}}(s). \quad (45)$$

Since (A_m, b) is controllable and $H(s)$ is asymptotically stable, then according to Lemma 2.2, there exists $c_o \in \mathbb{R}^n$ such that $c_o^\top H(s)$ is minimum phase with relatively degree 1. According to (41), we have $\theta^\top x(s) - \hat{r}(s) = \frac{c_o^\top\tilde{x}(s)}{c_o^\top H(s)} - k_m^\top e(s)$. Substituting this into (45) implies

$$\tilde{u}_{\mathrm{ref}}(s) = C(s)\left(\frac{c_o^\top\tilde{x}(s)}{c_o^\top H(s)} - k_m^\top e(s)\right) - \bar{c}^\top\bar{e}(s) + k_m^\top e(s) + (C(s)\theta^\top - k_m^\top)\tilde{x}_{\mathrm{ref}}(s) \quad (46)$$

Since $C(s)$ is stable, strictly proper and $c_o^\top H(s)$ is stable, minimum phase with relatively degree 1, we have that $\frac{C(s)c_o^\top}{c_o^\top H(s)}$ is BIBO stable and proper and therefore has a bounded $\mathcal{L}_1$ norm. So, equation (46) implies

$$\|\tilde{u}_{\mathrm{ref}_\tau}\|_{\mathcal{L}_\infty} \leq \|C(s)\theta^\top - k_m^\top\|_{\mathcal{L}_1}\|\tilde{x}_{\mathrm{ref}_\tau}\|_{\mathcal{L}_\infty} + \|\bar{c}^\top\|\|\bar{e}_\tau\|_{\mathcal{L}_\infty} + \|(1-C(s))k_m^\top\|_{\mathcal{L}_1}\|e_\tau\|_{\mathcal{L}_\infty} + \left\|\frac{C(s)c_o^\top}{c_o^\top H(s)}\right\|_{\mathcal{L}_1}\|\tilde{x}_\tau\|_{\mathcal{L}_\infty}.$$

for any $\tau \geq 0$. Using the result from Lemma 3.2, and substituting (44), (22) into this inequality implies (23). ∎

Proof: [Proof of Corollary 3.4] From (2), (3), (7) and (8), $u(s) = C(s)(\hat{r}(s) + k_g r(s)) + \bar{c}^\top\bar{e}(s) - k_m^\top x_d(s)$. Then

$$\|u_\tau\|_{\mathcal{L}_\infty} \leq L\|C(s)\|_{\mathcal{L}_1}\|x_{d_\tau}\|_{\mathcal{L}_\infty} + \|C(s)k_g\|_{\mathcal{L}_1}\|r_\tau\|_{\mathcal{L}_\infty} + \|\bar{c}\|\|\bar{e}_\tau\|_{\mathcal{L}_\infty} + \|k_m\|\|x_{d_\tau}\|_{\mathcal{L}_\infty}.$$

Notice that $\|x_{d_\tau}\|_{\mathcal{L}_\infty} \leq \|x_\tau\|_{\mathcal{L}_\infty}$. From Lemma 3.2, we have the upper bound in (19). Also, $\|\bar{e}\|_{\mathcal{L}_\infty} \leq \xi$ holds. Therefore

$$\|u_\tau\|_{\mathcal{L}_\infty} \leq (L\|C(s)\|_{\mathcal{L}_1} + \|k_m\|)\phi_2 + \|C(s)k_g\|_{\mathcal{L}_1}\|r\|_{\mathcal{L}_\infty} + \|\bar{c}\|\xi, \quad (47)$$

where ϕ_2 is defined in (21). Since this bound is independent of τ, we know this is a uniform bound on $\|u\|_{\mathcal{L}_\infty}$.

Let us consider $\frac{d}{dt}\|e(t)\|$ over $t \in (s_k, s_{k+1})$. According to (1), (22), (47), for any $t \in (s_k, s_{k+1})$,

$$\frac{d}{dt}\|e(t)\| \leq \|\dot{e}(t)\| = \|\dot{x}(t)\| \leq \|A - b\theta^\top\|\|x(t)\| + \|b\|\|u(t)\| \leq \psi_1$$

Solving the preceding inequality with the initial condition $\|e(s_k^+)\| = 0$, we obtain that for any $t \in (s_k, s_{k+1})$

$$\|e(t)\| \leq \psi_1(t - s_k). \quad (48)$$

Case I: if E_1 is violated, then $\|e(s_{k+1}^-)\| = \xi$. Using this in (48), we have $s_{k+1} - s_k \geq \frac{\xi}{\psi_1}$.

Case II: if E_2 is violated, then $\|x_d(s_{k+1}^-)e^\top(s_{k+1}^-)Pb\| = \xi^2$. Substituting this into (48), we have

$$\xi^2 = \|x_d(s_{k+1}^-)e^\top(s_{k+1}^-)Pb\| \leq \phi_2\|Pb\|\psi_1(s_{k+1} - s_k),$$

which implies $s_{k+1} - s_k \geq \frac{\xi^2}{\phi_2\psi_1\|Pb\|}$. Combining these two cases leads to (25).

Next, we derive the bound on $r_{i+1}^k - r_i^k$. From (8) we have

$$\bar{x}(s) = (s\mathbb{I} - \bar{A})^{-1}\bar{b}(\hat{r}(s) + k_g r(s)), \quad (49)$$

where $\hat{r}(s)$ is the Laplace transform of $\hat{r}(t) = \hat{\theta}^\top(t)x_d(t)$. The preceding equation implies that for any $\tau \geq 0$

$$\|\bar{x}_\tau\|_{\mathcal{L}_\infty} \leq L\|(s\mathbb{I} - \bar{A})^{-1}\bar{b}\|_{\mathcal{L}_1}\phi_2 + \|(s\mathbb{I} - \bar{A})^{-1}\bar{b}\|_{\mathcal{L}_1}\|k_g\|\|r\|_{\mathcal{L}_\infty}.$$

Since the bound above is uniform, and is independent of τ, it is also an upper bound on $\|\bar{x}\|_{\mathcal{L}_\infty}$.

Consider $\frac{d}{dt}\|\bar{e}(t)\|$ over $t \in (r_i^k, r_{i+1}^k)$. We have $\frac{d}{dt}\|\bar{e}(t)\| \leq \|\dot{\bar{e}}(t)\| = \|\dot{\bar{x}}(t)\| \leq= \psi_2$. Solving the preceding inequality with the initial condition $\|\bar{e}(r_i^{k+})\| = 0$ yields $\bar{e}(t) \leq \psi_2(t - r_i^k)$ for all $t \in (r_i^k, r_{i+1}^k)$. Since $\|\bar{e}(r_{i+1}^{k\,-})\| = \xi$, we have (26). ∎

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